Positive subreducts of MV-algebras

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MV-algebras extend the theory of Boolean algebras by replacing the two-element set of truth values $\{0, 1\}$ with the unit interval $[0, 1]$. They provide the algebraic semantics of Łukasiewicz many-valued logic. Inspired by the extensive study of bounded distributive lattices, which are the negation-free subreducts of Boolean algebras, we study the negation-free subreducts of MV-algebras. We call these algebras positive MV-algebras because all the terms are order-preserving in each argument. These algebras can be thought of as the many-valued version of bounded distributive lattices. We provide some results that can help to further develop the theory of these algebras:

1. positive MV-algebras are axiomatized by finitely many quasi-equations;
2. generalizing a result by Mundici for MV-algebras and lattice-ordered groups [4], positive MV-algebras are intervals of certain lattice-ordered monoids;
3. it is a standard result that any bounded distributive lattice L admits a unique Boolean algebra (called the free Boolean extension of L) in which it embeds so as to generate it as a Boolean algebra ([5, Thm. 4.1]); similarly, any positive MV-algebra admits a unique MV-algebra in which it embeds so as to generate it as an MV-algebra. (This is related to the fact that any equation in the language of MV-algebras is equivalent, for MV-algebras, to a system of equations in the language of positive MV-algebra.)

This talk is based on [1], [2, Ch. 4], and a joint work with P. Jipsen, T. Kroupa and S. Vannucci [3].

References

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