

- MARCO ABBADINI, VINCENZO MARRA, LUCA SPADA, *A duality for metrically complete lattice-ordered ordered groups.*

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Just like Boolean algebras are the algebraic semantics of classical propositional logic, Abelian lattice-ordered groups provide an algebraic tool for certain nonclassical logics. For example (due to their equivalence with MV-algebras), they are fundamental for the algebraic investigation of Łukasiewicz many-valued logic.

In a landmark paper in 1936, Stone obtained a representation theorem for Boolean algebras. In modern terms, his result amounts to a dual equivalence between Boolean algebras and Stone spaces. We provide an analogous duality for a class of Abelian lattice-ordered groups (with a designated element called unit). Our work takes inspiration from Yosida’s duality (1941) between compact Hausdorff spaces and metrically complete unital vector lattices. We extend Yosida’s theorem to metrically complete unital lattice-ordered groups, thus dropping the structure of real vector spaces. This calls for a generalised notion of compact Hausdorff space whose points carry an arithmetic character. The arithmetic character of a point is a metrically complete additive subgroup of \mathbb{R} containing 1—namely, either $\frac{1}{n}\mathbb{Z}$ for an integer $n = 1, 2, \dots$, or \mathbb{R} . The original Yosida duality is obtained by considering the full subcategory of spaces every point of which is assigned \mathbb{R} .

This presentation is based on [1].

[1] MARCO ABBADINI, VINCENZO MARRA, LUCA SPADA, *Stone-Gelfand duality for metrically complete lattice-ordered groups*, Submitted, preprint at arXiv/2210.15341.