▶ MARCO ABBADINI, *An algebraic version of Herbrand's theorem.*
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Herbrand's theorem is a fundamental result of mathematical logic, which allows a reduction of first-order logic to propositional logic. In its simplest form, it asserts that an existential statement $\exists x\, \alpha(x)$ with α quantifier-free is provable if and only if there are finitely many terms $c_1, \dots, c_n$ such that $\alpha(c_1) \vee \cdots \vee \alpha(c_n)$ is provable.

We provide an algebraic version of Herbrand's theorem. By "algebraic" we mean "in the setting of *first-order Boolean doctrines*", a notion that originates in Lawvere's work and can be seen as the algebraic semantics of classical first-order logic.

Our proof is semantic, i.e. we use models. One thing we find interesting is that our result gives Herbrand's theorem for the version of classical first-order logic whose semantics is compatible with admitting empty structures; it furthermore shows that Herbrand's theorem holds for many-sorted classical first-order logic. We use this result to show how to freely add one layer of quantifier alternation depth to a Boolean doctrine (which can be thought of as the algebra of quantifier-free formulas and which in general may lack quantifiers); this means that we show how to reduce inequalities between formulas with quantifier alternation depth at most 1 to a condition regarding inequalities of quantifier-free formulas.

This talk is based on a joint work with Francesca Guffanti [1].

[1] Marco Abbadini, Francesca Guffanti, *Freely adding one layer of quantifiers to a Boolean doctrine*, Submitted. Preprint avalaible at arXiv:2410.16328.