The algebraic structure of spaces of integrable functions

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Abstract. We characterize the functions $\mathbb{R}^I \to \mathbb{R}$ that preserve integrability, meaning that their pointwise application maps any I-tuple of integrable functions to an integrable function (as, for example, the sum $+\colon \mathbb{R}^2 \to \mathbb{R}$). We show that Dedekind σ-complete truncated vector lattices are precisely the algebras with integrability-preserving functions as function symbols and that satisfy all equations true in $\mathbb{R}$. We also show that an analogous study restricted to finite measure spaces gives the class of Dedekind σ-complete vector lattices with weak unit. Furthermore, we provide concrete models for free algebras in these categories.

Keywords: Integrable functions, Riesz space, vector lattice, weak unit, infinitary variety, equational classes, axiomatisation, free algebra, generation.